1. Výraz $ab+c$ vyjádři jako mnohočlen s proměnou *x*, který je uspořádaný sestupně, je-li:

a) $a=x+1 \qquad b=x^2-1 \qquad c=x^3+1$

$ab+c=(x+1)(x^2-1)+(x^3+1)=x^3-x+x^2-1+x^3+1=2x^3+x^2-x$

b) $a=2x-1 \qquad b=2x-1 \qquad c=x$

$ab+c=(2x-1)(2x-1)+(x)=4x^2-4x+1+x=4x^2-3x+1$

c) $a=b=c=3x-2$

$ab+c=(3x-2)(3x-2)+(3x-2)=9x^2-12x+4+3x-2=9x^2-9x+2$

d) $a=x-2 \qquad b=3-x \qquad c=x^2+1$

$ab+c=(x-2)(3-x)+(x^2+1)=3x-x^2-6+2x+x^2+1=5x-5=5(x-1)$

2. Urči mnohočlen, který je nutno přičíst k mnohočlenu $(x+y)^2+r^2$, abychom dostali mnohočlen $(x+y+r)^2$.

$(x+y)^2+r^2=x^2+2xy+y^2+r^2$

$(x+y+r)^2=(x+y+r)(x+y+r)=x^2+xy+xr+yx+y^2+yr+rx+ry+r^2=$

$=x^2+y^2+r^2+2xy+2xr+2yr$

$(x+y+r)^2-\left[(x+y)^2+r^2\right]=(x^2+y^2+r^2+2xy+2xr+2yr)-(x^2+2xy+y^2+r^2)=2xr+2yr$

Musíme připočíst výraz $2xr+2yr$.

3. Zjednoduš výraz:

a) $x(x^2+xy+y^2)-y(x^2-xy-y^2)-x(x^2+2y^2)=x^3+x^2y+xy^2-(x^2y-xy^2-y^3)-(x^3+2xy^2)=$
$=x^3+x^2y+xy^2-x^2y+xy^2+y^3-x^3-2xy^2=y^3$

b) $ab(c+d)-ac(b+d)+ad(b-c)+bc(a+d)-bd(a-c)+cd(a-b)=$
$=abc+abd-abc-acd+abd-acd+abc+bcd-abd+bcd+acd-bcd=$
$=abc+abd-acd+bcd$

c) $4a(5b-2a)-4(7a^2-3ab)-2a(3a-3b)=20ab-8a^2-28a^2+12ab-6a^2+6ab=$
$=38ab-42a^2=2a(19b-21a)$

d) $1,4x(0,5x-0,3y)-5(0,4y^2-4xy)+0,2y(8y-5x)=0,7x^2-0,42xy-2y^2+20xy+1,6y^2-xy=$
$=0,7x^2-18,58xy-0,4y^2$

e) $r^3(r^2+3)+r^2(r^3+r^2)-r^3(r+1)=r^5+3r^3+r^5+r^4-r^4-r^3=2r^5+2r^3=2r^3(r^2+1)$

f) $x^3(x+y^3)-(xy)^3+(2x^2)^3=x^4+x^3y^3-x^3y^3+8x^6=x^4+8x^6=x^4(1+8x^2)$

g) $(a^2b^3)^2+(2a^2)^2y^2-(a^2y)^2-a^4(b^6+1)=a^4b^6+4a^4y^2-a^4y^2-a^4b^6-a^4=3a^4y^2-a^4=$
$=a^4(3y^2-1)$

4. Zjednoduš výraz:

a) $(2x-1)^3-(x-2)^3=(8x^3-12x^2+6x-1)-(x^3-6x^2+12x-8)=7x^3-6x^2-6x+7$

b)
$$(3x+y)^3-(9x^2+6xy+y^2)(3x-y)=$$
$$=27x^3+27x^2y+9xy^2+y^3-(27x^3+18x^2y+3xy^2-9x^2y-6xy^2-y^3)=$$
$$=18x^2y+12xy^2+2y^3$$

c) $(a+2)^3-3(a+2)^2(a+1)+3(a+2)(a+1)^2-(a+1)^3=[(a+2)-(a+1)]^3=(1)^3=1$

d)
$$(a^2-1)^3-(a^2-1)(a^2+1)^2+2a^2(a^2-2)+a^4(a^4+2)=$$
$$=(a^6-3a^4+3a^2-1)-(a^2-1)(a^4+2a^2+1)+2a^4-2a^2+a^8+2a^4=$$
$$=a^8+a^6+a^4+a^2-1-(a^6+2a^4+a^2-a^4-2a^2-1)=a^8$$

e) $(2x-1)^3(2x+1)^3=[(2x-1)(2x+1)]^3=[4x^2-1]^3=64x^6-48x^4+12x^2-1$

f)
$$(a^2-ab+b^2)^3(a+b)^3=[(a^2-ab+b^2)(a+b)]^3=[a^3+a^2b-a^2b-ab^2+b^2a+b^3]^3=$$
$$=(a^3+b^3)^3$$

g)
$$(a+b)^2-(a-b)^2+(ab+1)^2-(ab-1)^2=$$
$$=a^2+2ab+b^2-(a^2-2ab+b^2)+a^2b^2+2ab+1-(a^2b^2-2ab+1)=$$
$$=4ab+4ab=8ab$$

h) $\left[(p+1)^2-(p-1)^2\right]^2=\left[p^2+2p+1-(p^2-2p+1)\right]^2=(4p)^2=16p^2$

i)
$$\left[(2x^2-3y^3)^2+(3x^2+2y^3)^2\right]^2=\left[4x^4-12x^2y^3+9y^6+(9x^4+12x^2y^3+4y^6)\right]^2=$$
$$=(13x^4+13y^6)^2=\left[13(x^4+y^6)\right]^2=13^2(x^8+2x^4y^6+y^{12})=169x^8+338x^4y^6+169y^{12}$$

5. O kolik se zvětší hodnota výrazu $(a+b+1)^2$, zvětší-li se číslo a o 1?

Spočtu výraz

$$([a+1]+b+1)^2-(a+b+1)^2=(a+b+2)^2-(a+b+1)^2=(a+b+2)(a+b+2)-(a+b+1)(a+b+1)=$$
$$=a^2+ab+2a+ba+b^2+2b+2a+2b+4-(a^2+ab+a+ba+b^2+b+a+b+1)=$$
$$=a^2+2ab+b^2+4a+4b+4-(a^2+2ab+b^2+2a+2b+1)=2a+2b+3$$

6. Stanov podmínky a děl:

a)
$$\begin{array}{l}(6x^2-11x-10):(3x+2)=2x-5\\ -(6x^2+4x)\\ \qquad\quad -15x-10\\ \qquad -(-15x-10)\\ \qquad\qquad\quad 0\end{array}$$

b)

$$\left(a^3-b^3\right):\left(a-b\right)=a^2+ab+b^2$$
$$-\left(a^3-a^2b\right)$$
$$a^2b-b^3$$
$$-\left(-a^2b-ab^2\right)$$
$$ab^2-b^3$$
$$-\left(ab^2-b^3\right)$$
$$0$$

c)

$$\left(c^3+c^2-11c-15\right):\left(c+3\right)=c^2-2c-5$$
$$-\left(c^3+3c^2\right)$$
$$-2c^2-11c-15$$
$$-\left(-2c^2-6c\right)$$
$$-5c-15$$
$$-\left(-5c-15\right)$$

d)

$$\left(9y^4+26y^2+25\right):\left(3y^2-2y+5\right)=3y^2+2y+5$$
$$-\left(9y^4-6y^3+15y^2\right)$$
$$6y^3+11y^2+25$$
$$-\left(6y^3-4y^2+10y\right)$$
$$15y^2-10y+25$$
$$-\left(15y^2-10y+25\right)$$

e)

$$\left(x^4-8x^3+16x^2-7x-2\right):\left(x^2-3x+2\right)=x^2-5x-1$$
$$-\left(x^4-3x^3+2x^2\right)$$
$$-5x^3+14x^2-7x-2$$
$$-\left(-5x^3+15x^2-10x\right)$$
$$-x^2+3x-2$$
$$-\left(-x^2+3x-2\right)$$

f)

$$\left(11p^3-32+19p^2+3p^4-28p\right):(4-3p)=$$

$$\left(3p^4+11p^3+19p^2-28p-32\right):(-3p+4)=-p^3-5p^2-13p-8$$

$$-\left(3p^4-4p^3\right)$$

$$15p^3+19p^2-28p-32$$

$$-\left(15p^3-20p^2\right)$$

$$39p^2-28p-32$$

$$-\left(39p^2-52p\right)$$

$$24p-32$$

$$-\left(24p-32\right)$$

g)

$$\left(x^5+4x^4+4x^3-x-2\right):(x+2)=x^4+2x^3-1$$

$$-\left(x^5+2x^4\right)$$

$$2x^4+4x^3-x-2$$

$$-\left(2x^4+4x^3\right)$$

$$-x-2$$

$$-(-x-2)$$

h)

$$\left(x^7-x^5-x^4+1\right):\left(x^2-1\right)=x^5-x^2-1$$

$$-\left(x^7-x^5\right)$$

$$-x^4+1$$

$$-\left(-x^4+x^2\right)$$

$$-x^2+1$$

$$-\left(-x^2+1\right)$$

i)

$$\left(x^8+x^7-x^6+x^5+x^3+x^2-x+1\right):\left(x^5+1\right)=x^3+x^2-x+1$$

$$-\left(x^8\qquad\qquad\qquad+x^3\right)$$

$$x^7-x^6+x^5+x^2-x+1$$

$$-\left(x^7\qquad\qquad+x^2\right)$$

$$-x^6+x^5-x+1$$

$$-\left(-x^6\qquad-x\right)$$

$$x^5+1$$

$$-\left(x^5+1\right)$$

j)

$$\left(3y^4-4y^3-7y^2+8y+2\right):\left(4y^2-8\right)=\frac{3}{4}y^2-y-\frac{1}{4}$$
$$-\left(3y^4 \qquad -6y^2\right)$$
$$-4y^3-y^2+8y+2$$
$$-\left(-4y^3 \qquad +8y\right)$$
$$-y^2 \qquad +2$$
$$-\left(-y^2 \qquad +2\right)$$
$$0$$

k)

$$\left(2x^4-7x^3-2x^2+10x\right):\left(2x^2-3x+2\right)=x^2-2x-5+\frac{-x+10}{2x^2-3x+2}$$
$$-\left(2x^4-3x^3+2x^2\right)$$
$$-4x^3-4x^2+10x$$
$$-\left(-4x^3+6x^2-4x\right)$$
$$-10x^2+14x$$
$$-\left(-10x^2+15x-10\right)$$
$$-x+10$$

7. Rozlož mnohočleny na součin:

a) $x(a+b)^2+x^2(a+b)=x(a+b)\left[(a+b)+x\right]=x(a+b)(a+b+x)$

b) $ax^5-2a^2x^4+a^3x^3=ax^3\left(x^2-2ax+a^2\right)=ax^3(x-a)^2$

c) $8b^2-18c^2=2\left(4b^2-9c^2\right)=2\left[(2b)^2-(3c)^2\right]=2(2b-3c)(2b+3c)$

d) $9p^4(a-b)-25q^2(a-b)=(a-b)\left[\left(3p^2\right)^2-(5q)^2\right]=(a-b)\left(3p^2+5q\right)\left(3p^2-5q\right)$

e)
$$9x^2-6xy+y^2-z^2=\left[(3x)^2-6xy+y^2\right]-z^2=(3x-y)^2-z^2=\left[(3x-y)-z\right]\left[(3x-y)+z\right]=$$
$$=(3x-y-z)(3x-y+z)$$

f) $(a-b)x^4+(b-a)x^2=x^2\left[(a-b)x^2-(a-b)\right]=(a-b)x^2\left[x^2-1\right]=(a-b)x^2(x+1)(x-1)$

g)
$$\left(a^2+b^2-c^2\right)^2-4a^2b^2=\left(a^2+b^2-c^2\right)^2-(2ab)^2=\left(a^2+b^2-c^2-2ab\right)\left(a^2+b^2-c^2+2ab\right)=$$
$$=\left(a^2-2ab+b^2-c^2\right)\left(a^2+2ab+b^2-c^2\right)=\left[(a-b)^2-c^2\right]\left[(a+b)^2-c^2\right]=$$
$$=\left[(a-b)-c\right]\left[(a-b)+c\right]\left[(a+b)-c\right]\left[(a+b)+c\right]=$$
$$=(a-b-c)(a-b+c)(a+b-c)(a+b+c)$$

h) $2a^5-2a=2a\left(a^4-1\right)=2a\left[\left(a^2\right)^2-1\right]=2a\left(a^2-1\right)\left(a^2+1\right)=2a(a-1)(a+1)\left(a^2+1\right)$

i) $$\begin{aligned}&a(p-q+1)(ax^2+b)+b(p-q+1)(bx^2-a)+2abx^2(p-q+1)=\\&=(p-q+1)\left[a(ax^2+b)+b(bx^2-a)+2abx^2\right]=(p-q+1)\left[a^2x^2+ab+b^2x^2-ba+2abx^2\right]=\\&=(p-q+1)\left[a^2x^2+b^2x^2+2abx^2\right]=(p-q+1)x^2(a^2+2ab+b^2)=\\&=(p-q+1)x^2(a+b)^2\end{aligned}$$

j) $$\begin{aligned}&(r+s)^4-r^4=\left[(r+s)^2\right]^2-\left[r^2\right]^2=\left[(r+s)^2+r^2\right]\left[(r+s)^2-r^2\right]=\\&=\left[r^2+rs+s^2+r^2\right]\left[(r+s)-r\right]\left[(r+s)+r\right]=(2r^2+rs+s^2)s(2r+s)\end{aligned}$$

k) $$\begin{aligned}&xz-yz-x^2+2xy-y^2=z(x-y)-(x^2-2xy+y^2)=z(x-y)-(x-y)^2=\\&=(x-y)\left[z-(x-y)\right]=(x-y)(z-x+y)\end{aligned}$$

l) $x^3-3x^2-4x+12=x^2(x-3)-4(x-3)=(x-3)(x^2-4)=(x-3)(x-2)(x+2)$

m) $$\begin{aligned}&2k^4-k^3+k-2=2k^4-2-k^3+k=2(k^4-1)-k(k^2-1)=2\left(\left[k^2\right]^2-1\right)-k(k^2-1)=\\&=2(k^2-1)(k^2+1)-k(k^2-1)=(k^2-1)\left[2(k^2+1)-k\right]=(k-1)(k+1)(2k^2-k+2)\end{aligned}$$

n) $$\begin{aligned}&y^4-2y^3+2y^2-2y+1=y^4-2y^3+y^2+y^2-2y+1=y^2(y^2-2y+1)+(y^2-2y+1)=\\&=y^2(y-1)^2+(y-1)^2=(y-1)^2(y^2+1)=(y-1)(y+1)(y^2+1)\end{aligned}$$

o) $$\begin{aligned}&2h^2+h-1=h^2+h+h^2-1=h(h+1)+(h^2-1)=h(h+1)+(h+1)(h-1)=(h+1)\left[h+(h-1)\right]=\\&=(h+1)(2h-1)\end{aligned}$$

p) $27r^4-r=r(27r^3-1)=r\left[(3r)^3-1\right]=r(3r-1)(9r^2+3r+1)$

q) $$\begin{aligned}&a^3+3a^2+4a+2=a^3+3a^2+2a+2a+2=a(a^2+3a+2)+2(a+1)=a(a+1)(a+2)+2(a+1)=\\&=(a+1)\left[a(a+2)+2\right]=(a+1)(a^2+2a+2)\end{aligned}$$

r) $x^2-x-72=x^2-9x+8x-72=x(x-9)+8(x-9)=(x-9)(x+8)$

s) $$\begin{aligned}&x^3+x^2-42x=x(x^2+x-42)=x\left[x^2+7x-6x-42\right]=x\left[x(x+7)-6(x+7)\right]=\\&=x(x+7)(x-6)\end{aligned}$$

t) $4x^2-8x+3=4x^2-6x-2x+3=2x(2x-3)-(2x-3)=(2x-3)(2x-1)$

u) $3a^2+5a-2=3a^2+6a-a-2=3a(a+2)-(a+2)=(a+2)(3a-1)$

v) $2y^2+3y+1=2y^2+2y+y+1=2y(y+1)+(y+1)=(y+1)(2y+1)$

w) $x^2+(a-3)x+2(1-a)=x^2+ax-3x-2a+2=x^2+ax-x-2x-2a+2=$
$=x(x+a-1)-2(x+a-1)=(x+a-1)(x-2)$

x) $x^6-y^6=\left(x^3\right)^2-\left(y^3\right)^2=\left(x^3-y^3\right)\left(x^3+y^3\right)=(x-y)\left(x^2+xy+y^2\right)(x+y)\left(x^2-xy+y^2\right)$

y) $\left(x^2-2x+3\right)^2-\left(x^2-2x-3\right)^2=\left[\left(x^2-2x+3\right)-\left(x^2-2x-3\right)\right]\left[\left(x^2-2x+3\right)+\left(x^2-2x-3\right)\right]=$
$6\left(2x^2-4x\right)=6\cdot 2x(x-2)=12x(x-2)$

z) $2x^4+x^3+4x^2+x+2=2x^4+x^3+2x^2+2x^2+x+2=x^2\left(2x^2+x+2\right)+\left(2x^2+x+2\right)=$
$=\left(2x^2+x+2\right)\left(x^2+1\right)$

ž) $t^3+3t^2+4t+2=t^3+3t^2+2t+2t+2=t\left(t^2+3t+2\right)+2(t+1)=t(t+1)(t+2)+2(t+1)=$
$=(t+1)\left[t(t+2)+2\right]=(t+1)\left(t^2+2t+2\right)$